Rational and Irrational Numbers

Write each fraction as a decimal.

1. \( \frac{1}{8} \)  
2. \( \frac{9}{16} \)  
3. \( \frac{11}{20} \)  
4. \( \frac{5}{8} \)

5. \( \frac{14}{15} \)  
6. \( \frac{2}{12} \)  
7. \( \frac{3}{100} \)  
8. \( \frac{16}{5} \)

Find the two square roots of each number.

9. 25  
10. 1  
11. \( \frac{25}{4} \)  
12. \( \frac{121}{49} \)

Find the cube root of each number.

13. 8  
14. 216  
15. 1  
16. 2197

Approximate each irrational number to the nearest hundredth without using a calculator.

17. \( \sqrt{32} \)  
18. \( \sqrt{118} \)  
19. \( \sqrt{18} \)  
20. \( \sqrt{319} \)

Approximate each irrational number to the nearest hundredth without using a calculator. Then plot each number on a number line.

21. \( \sqrt{8} \)  
22. \( \sqrt{75} \)

23. A tablet weighs 1.23 pounds. What is its weight written as a mixed number?

24. The area of a square mirror is 256 in\(^2\). A rectangular mirror has a width the same as the square mirror’s width. Its length is two inches longer than its width. What is the area of the rectangular mirror?
Rational and Irrational Numbers

Practice and Problem Solving: C

Solve.

1. One nickel is \( \frac{39}{500} \) inch thick. Fifteen nickels are stacked vertically. How many inches tall is the stack? Give your answer as a decimal.

_________________________________________________________________________________________

2. One quarter is \( \frac{191}{200} \) inch in diameter. Eight quarters are placed side-by-side along a line. How many inches long is the line of quarters? Give your answer as a decimal.

_________________________________________________________________________________________

3. Is \( \frac{41}{50} \) closer to \( \frac{9}{11} \) or \( \frac{10}{11} \)? Verify your answer.

_________________________________________________________________________________________

_________________________________________________________________________________________

Find the two square roots of each number. (Hint: First write the decimal as a fraction.)

4. 0.25 \( \underline{\quad} \) 5. 0.0625 \( \underline{\quad} \) 6. 0.4 \( \underline{\quad} \)

Approximate each irrational number to the nearest hundredth without using a calculator. Then plot each lettered point on the number line.

7. A: \( \sqrt{3} \) \( \underline{\quad} \) 8. B: \( \sqrt{18} \) \( \underline{\quad} \)

Answer the questions below.

9. How does finding a cube root differ from finding a square root of a positive integer? How do the answers differ?

_________________________________________________________________________________________

_________________________________________________________________________________________

10. Each page of a photo album holds 3 rows of 4 square photos. The area of each photo is 25 cm\(^2\). There is 2 cm space between photos and a 3 cm border around the group of pictures. What are the dimensions of one page of the photo album?

_________________________________________________________________________________________
Rational and Irrational Numbers

Practice and Problem Solving: D

Write each fraction as a decimal. The first one is done for you.

1. \( \frac{1}{9} \) \( \rightarrow \) \( 0.1 \)
2. \( \frac{11}{20} \)
3. \( \frac{9}{16} \)

Write each decimal as a fraction in simplest form. The first one is done for you.

4. 0.258
5. 4.8
6. 0.333

\( \frac{258}{1000} = \frac{129}{500} \)

Find the two square roots of each number. The first one is done for you.

7. 16
8. 49
9. \( \frac{25}{4} \)

\( 4, -4 \)

Find the cube root of each number. The first one is done for you.

10. 343
11. 1
12. \( \frac{8}{27} \)

\( 7 \times 7 \times 7 = 343 \)

Approximate each irrational number to the nearest hundredth without using a calculator. The first one is done for you.

13. \( \sqrt{32} \) \( \approx \) 5.66
14. \( \sqrt{59} \)
15. \( \sqrt{118} \)

Solve.

16. The world’s smallest country is Vatican City. It covers \( \frac{17}{100} \) square mile. What is Vatican City’s area written as a decimal?

17. A square sandbox has an area of 25 ft\(^2\). What is the length of each of its sides? (Hint: side = \( \sqrt{25} \) )
Rational and Irrational Numbers

Reteach

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal may terminate. A decimal may repeat.

\[
\begin{align*}
\frac{3}{4} = & \quad 0.75 \\
4 \overline{)3.00} \\
-28 \downarrow \\
20 \\
-20 \\
0 \\
\end{align*}
\]

\[
\begin{align*}
\frac{1}{3} = & \quad 0.\overline{3} \\
3 \overline{)1.00} \\
-9 \downarrow \\
10 \\
-9 \\
1 \\
\end{align*}
\]

Complete to write each fraction as a decimal.

1. \[\frac{15}{4} = 4)15.00\] 2. \[\frac{5}{6} = 6)5.00\] 3. \[\frac{11}{3} = 3)11.00\]

Every positive number has two square roots, one positive and one negative.

Since \(5 \times 5 = 25\) and also \(-5 \times -5 = 25\), \(\sqrt{25} = 5\) and \(-\sqrt{25} = -5\), both 5 and -5 are square roots of 25.

Every positive number has one cube root.

Since \(4 \times 4 \times 4 = 64\), 4 is the cube root of 64.

Find the two square roots for each number.

4. 81
5. 49
6. \[\frac{25}{36}\]

Find the cube root for each number.

7. 27
8. 125
9. 729
Rational and Irrational Numbers

Reading Strategies: Compare and Contrast

**Are Real Numbers**
- Can be written as a fraction.
- Cannot be written as a fraction.

**Rational Numbers**
- Examples:
  - $8 = \frac{8}{1}$
  - $-2.5 = -\frac{5}{2}$
  - $\frac{49}{81} = \frac{7}{9}$
  - $-\frac{49}{81} = -\frac{7}{9}$
  - $0.\overline{6} = \frac{2}{3}$
  - $0.375 = \frac{3}{8}$
- Decimals terminate or repeat.

**Irrational Numbers**
- Examples:
  - $\sqrt{2} = 1.414213...$
  - $\pi = 3.141592...$
  - $\sqrt{24} = 4.8989794...$
- Can be written as a decimal.
- Decimals are infinite and nonrepeating.

**Use the chart to answer the following questions.**

1. Is 0.62 a rational number? Why or why not?

2. Is $\sqrt{7}$ a rational number? Why or why not?

3. Can an irrational number be a decimal? If so, give an example.

4. Can a rational number be a repeating decimal? If so, give an example.

5. What kind of decimal is an irrational number? Give an example.

6. What do rational and irrational numbers have in common?
Problem 1

Think about decimal equivalents of common fractions to rewrite \( \frac{2}{3} \) as a decimal.

\[
\begin{align*}
\frac{1}{2} &= 0.5 \\
\frac{1}{4} &= 0.25 \\
\frac{1}{3} &= 0.3 \\
\frac{3}{4} &= 0.75 \\
\frac{2}{3} &= 0.\overline{6}
\end{align*}
\]

So, \( \frac{2}{3} = 0.\overline{6} \).

Problem 2

Think: What number times itself equals 81?

\[
\sqrt{81} = 9 \\
-\sqrt{81} = -9
\]

Problem 3

\[
\sqrt{700} \approx 26.5
\]

It reads “the square root of 700 is about 26.5.”

1. Which decimal equivalent of a common fraction would you use to rewrite \( 1 \frac{1}{4} \) as a decimal?

_________________________________________________________________________________________

2. Why is \( 5^2 \) read as “five squared”?

_________________________________________________________________________________________

3. Why do you use the term “about” when reading the answer to Problem 3 above?

_________________________________________________________________________________________
List all number sets that apply to each number.

1. \( -\frac{4}{5} \)

2. \( \sqrt{15} \)

3. \( -2 \)

4. \( -25 \)

5. \( 0.\overline{3} \)

6. \( \frac{20}{4} \)

Tell whether the given statement is true or false. Explain your choice.

7. All real numbers are rational.

8. All whole numbers are integers.

Identify the set of numbers that best describes each situation.

9. the amount of money in a bank account

10. the exact temperature of a glass of water in degrees Celsius

Place each number in the correct location on the Venn diagram.

11. \( -\frac{5}{9} \)

12. \( -\sqrt{100} \)

13. \( \pi \)

14. \( \sqrt{25} \)
List all number sets that apply to each number.

1. \(-\sqrt{36}\)
   - Rational
   - Real

2. \(-\frac{16}{2}\)
   - Rational
   - Real

3. 0.125185623
   - Rational
   - Real

4. \(\frac{\sqrt{25}}{5}\)
   - Rational
   - Real

5. \(\frac{18}{19}\)
   - Rational
   - Real

6. \(\frac{4}{5} \cdot \frac{10}{4}\)
   - Rational
   - Real

Identify the set of numbers that best describes each situation. Explain your choice.

7. the possible scores in a card game in which points are added or deducted after each hand
   - Integers
   - Real
   - Rational

8. elevation of land compared to sea level
   - Integers
   - Real
   - Rational

Answer each question.

9. Is it possible to count the number of rational numbers there are between any two integers?
   - No, there are infinitely many rational numbers between any two integers.

10. If you take the square root of every whole number from 1 through 100, how many of them will be whole numbers? How many will be irrational numbers?
    - There will be 10 whole numbers (the perfect squares), and 90 irrational numbers.

11. What numbers are integers but not whole numbers?
    - Fractions and decimals

12. What negative numbers are not integers?
    - Rational numbers
List all number sets that apply to each number. The first one is done for you.

1. $\frac{1}{2}$
   - real, rational

2. $\sqrt{3}$

3. 0.9
   - rational

4. $-3$
   - integer

5. 0.6
   - decimal

6. 18
   - whole number

Tell whether the given statement is true or false. Explain your choice.

7. All fractions are real numbers.
   - True. All fractions are real numbers.

8. All negative numbers are integers.
   - False. Not all negative numbers are integers. For example, $\frac{-1}{2}$ is a negative fraction but not an integer.

Identify the set of numbers that best describes each situation. Explain your choice.

9. The number of people in a movie theater
   - Whole numbers
   - People cannot be counted in fractions or decimals in the context of whole numbers.

10. Roll a pair of number cubes and take the square root of the sum
    - Irrational numbers
    - The square root of a sum of two integers can result in an irrational number.

Place each of the given numbers in the correct location on the Venn diagram. The first one is done for you.

11. $\frac{2}{3}$
    - Rational Numbers
    - Integers

12. $-99$
    - Integers
    - Whole Numbers

13. $\frac{10}{11}$
    - Rational Numbers
    - Whole Numbers

14. 1,000
    - Whole Numbers
    - Real Numbers

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Numbers can be organized into groups. Each number can be placed into one or more of the groups.

**Real numbers** include all rational and irrational numbers. All of the numbers that we use in everyday life are real numbers.

- If a real number can be written as a fraction, it is a **rational number**.
  - If it cannot be written as a fraction, it is an **irrational number**.
- If a rational number is a whole number, or the opposite of a whole number, then it is an **integer**.
- If an integer is positive or 0, then it is a **whole number**.

You can use these facts to categorize any number.

**A. What kind of number is 10?**

- Is it a real number? **Yes.**
- Is it a rational number? Can it be written as a fraction? **Yes:** \( \frac{10}{1} \)
- Is it an integer? Is it a whole number or the opposite of a whole number? **Yes.**
- Is it a whole number? **Yes.**
- So 10 is a real number, a rational number, an integer, and a whole number.

**B. What kind of number is \( \frac{\sqrt{9}}{3} \)?**

- Is it a real number? **Yes.**
- Is it a rational number? Can it be written as a fraction? **No.** \( \frac{\sqrt{9}}{3} \) simplifies to 3. If you try to find the square root of 3, you will get a decimal answer that goes on forever but does not repeat: 1.7320508… This cannot be written as a fraction.
- So \( \frac{\sqrt{9}}{3} \) is a real, irrational number.

**Answer each question to identify the categories the given number belongs to.**

\( \sqrt{16} \)

1. Is it a real number? _________________
2. Is it a rational number? Can it be written as a fraction? _________________
3. Is it an integer? Is it a whole number or the opposite of a whole number? _________________
4. Is it a whole number? _________________
5. List all of the categories \( \sqrt{16} \) belongs to. _________________
Sets of Real Numbers

Reading Strategies: Use a Venn Diagram

1. A real number is a ____________ or an ____________ number.

2. A rational number can be written as a ____________ or a ____________.

3. Both ____________ and ____________ decimals are rational numbers.

4. A set of integers is the set of ____________ and ____________ whole numbers and zero.

5. The whole numbers are the set of ____________ numbers and zero.

6. Place each number on the proper line on the Venn diagram.
   a. −5  b. 0.34  c. 11  d. π
Problem 1

Classify each number. Use the flowchart to help you.

1. \( \sqrt{15} \) __________
2. \( \frac{3}{0} \) __________
3. \( \frac{1}{9} \) __________
4. \(-13\) __________
Ordering Real Numbers

Practice and Problem Solving: A/B

Compare. Write <, >, or = .

1. \( \sqrt{5} + 3 \) \( \bigcirc \) \( \sqrt{5} + 4 \)
2. \( \sqrt{6} + 13 \) \( \bigcirc \) \( \sqrt{10} + 13 \)
3. \( \sqrt{7} + 4 \) \( \bigcirc \) \( 5 + \sqrt{6} \)
4. \( 8 + \sqrt{2} \) \( \bigcirc \) \( \sqrt{8} + 2 \)
5. \( 3 + \sqrt{3} \) \( \bigcirc \) \( \sqrt{13} - 7 \)
6. \( 11 - \sqrt{3} \) \( \bigcirc \) \( 5 - \sqrt{3} \)

Use the table to answer the questions.

7. List the butterflies in order from greatest to least wingspan.

<table>
<thead>
<tr>
<th>Butterfly</th>
<th>Wingspan (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great white</td>
<td>3.75</td>
</tr>
<tr>
<td>Large orange sulphur</td>
<td>( 3 \frac{3}{8} )</td>
</tr>
<tr>
<td>Apricot sulphur</td>
<td>2.625</td>
</tr>
<tr>
<td>White-angled sulphur</td>
<td>3.5</td>
</tr>
</tbody>
</table>

8. The pink-spotted swallowtail’s wingspan can measure \( 3 \frac{5}{16} \) inches.
   Between which two butterflies should the pink-spotted swallowtail be in your list from question 7?

Order each group of numbers from least to greatest.

9. \( \sqrt{8} \), 2, \( \frac{\sqrt{7}}{2} \)
10. \( \sqrt{12} \), \( \pi \), 3.5

11. \( \sqrt{26} \), \(-20\), 13.5, \( \sqrt{35} \)
12. \( \sqrt{6} \), \(-5.25\), \( \frac{3}{2} \), 5

Solve.

13. Four people have used different methods to find the height of a wall. Their results are shown in the table. Order their measurements from greatest to least. \( \pi \approx 3.14 \)

<table>
<thead>
<tr>
<th>Wall Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allie</td>
</tr>
<tr>
<td>Byron</td>
</tr>
<tr>
<td>Justin</td>
</tr>
<tr>
<td>Rosa</td>
</tr>
<tr>
<td>( \sqrt{12} - 1 )</td>
</tr>
<tr>
<td>( \frac{5}{2} )</td>
</tr>
<tr>
<td>2.25</td>
</tr>
<tr>
<td>( 1 + \frac{\pi}{2} )</td>
</tr>
</tbody>
</table>
1. Order $4.6, \sqrt{13} + 1$, and $2\pi - 1.68$ from least to greatest. Use $\pi \approx 3.14$.
   a. From least to greatest, the numbers are:
   
   ____________________________________________________________
   
   b. Would the order change if you used $\pi \approx \frac{22}{7}$? Explain.
   
   ____________________________________________________________

2. Four people are using different methods to measure the width of shelves to be installed in a closet using 3.5-centimeter brackets. Their results are shown in the table.

<table>
<thead>
<tr>
<th>Shelf Width (m)</th>
<th>Allie</th>
<th>Byron</th>
<th>Justin</th>
<th>Rosa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{12} - 2.2$</td>
<td>$\frac{\sqrt{23}}{2} - 1$</td>
<td>1.18</td>
<td>$1 + \frac{\pi}{9}$</td>
<td></td>
</tr>
</tbody>
</table>

   a. Order their measurements from greatest to least.
   
   ____________________________________________________________
   
   b. The width of the closet, 1.2 meters, is shown on the number line. Graph the four measurements shown in the table.
   
   ____________________________________________________________
   
   c. Whose shelf or shelves would be suitable to use in the closet? Explain.
   
   ____________________________________________________________
   
   ____________________________________________________________
   
   ____________________________________________________________
Ordering Real Numbers

Compare. Write <, >, or =. The first one is done for you.
1. \( \sqrt{2} + 1 \) \( \circledless \) \( \sqrt{2} + 8 \)
2. \( \sqrt{2} + 5 \) \( \bigcirc \) \( \sqrt{2} + 3 \)
3. \( \sqrt{3} + 5 \) \( \bigcirc \) \( 5 + \sqrt{6} \)
4. \( 8 + \sqrt{2} \) \( \bigcirc \) \( \sqrt{8} + 2 \)
5. \( 3 + \sqrt{3} \) \( \bigcirc \) \( \sqrt{7} - 3 \)
6. \( 5 - \sqrt{3} \) \( \bigcirc \) \( - \sqrt{3} + 5 \)

Graph the numbers on the number line. Then order them from least to greatest.
7. \( \sqrt{2}, \pi, 4.5 \)

From least to greatest, the numbers are ______, ______, and ______.

Order the numbers from least to greatest. The first one is done for you.
8. \( 2, \frac{\sqrt{2}}{2}, -10 \)

\( -10, \frac{\sqrt{2}}{2}, 2 \)

9. \( 7, \pi, \sqrt{3} \)

10. \( \sqrt{8}, -4, 1.5 \)

11. \( \sqrt{6}, -5.5, \frac{3}{2} \)

Solve.
12. Four people have measured the height of a wall using different methods. Their results are shown in the table. Order their measurements from least to greatest.

<table>
<thead>
<tr>
<th>Wall Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allie</td>
</tr>
<tr>
<td>( \sqrt{8} )</td>
</tr>
</tbody>
</table>
Ordering Real Numbers

Reteach

Compare and order real numbers from least to greatest.

Order $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ from least to greatest.

You can use a calculator to approximate irrational numbers.

$\sqrt{22} \approx 4.69$

You know that $\pi \approx 3.14$, so you can find the approximate value of $\pi + 1$.

$\pi + 1 \approx 3.14 + 1 \approx 4.14$

Plot $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ on a number line.

On a number line, the values of numbers increase as you move from left to right. So, to order these numbers from least to greatest, list them from left to right.

$\pi + 1$, $4\frac{1}{2}$, and $\sqrt{22}$

Order each group of numbers from least to greatest.

1. $4$, $\pi$, $\sqrt{8}$

2. $5$, $\frac{17}{3}$, $\pi + 2$

3. $\sqrt{2}$, $1.7$, $-2$

4. $2.5$, $\sqrt{5}$, $\frac{3}{2}$

5. $3.7$, $\sqrt{13}$, $\pi + 1$

6. $\frac{5}{4}$, $\pi - 2$, $\frac{\sqrt{5}}{2}$
Ordering Real Numbers

Reading Strategies: Connect Words with Symbols

To compare real numbers, you can use the symbols <, >, and =.

To approximate irrational numbers, you can use the symbol ≈.

The symbol < means “less than.”

\[
\frac{1}{2} < 2 \quad \text{Read as “} \frac{1}{2} \text{ is less than 2.”}
\]

The symbol > means “greater than”:

\[
\sqrt{6} > \sqrt{5} \quad \text{Read as “The square root of 6 is greater than the square root of 5.”}
\]

The symbol = means “equal to”:

\[
\sqrt{16} = 4 \quad \text{Read as “The square root of 16 is equal to 4” OR “The square root of 16 equals 4.”}
\]

The sign ≈ means “approximately equal to”:

\[
\pi \approx 3.14 \quad \text{Read as “} \pi \text{ is approximately equal to 3.14.” OR “} \pi \text{ is approximately 3.14.”}
\]

Write in words.

1. \[\sqrt{13} < 4\]

2. \[0.501 \approx \frac{1}{2}\]

3. \[\sqrt{25} = 5\]

4. \[\pi + 1 > \frac{2}{3}\]

Write using symbols.

5. Eighteen-halves is equal to nine. \[\frac{18}{2} = 9\]

6. 5.17 is greater than the square root of twenty-three. \[5.17 > \sqrt{23}\]

7. Two-thirds is less than pi. \[\frac{2}{3} < \pi\]
Problem 1

Compare. Write $<$, $>$, or $=$.

\[
\sqrt{5} + 2 \bigcirc 5 + \sqrt{2}
\]

\[
\sqrt{5} \approx 2.2 \quad \sqrt{2} \approx 1.4
\]

Substitute.

\[
2.2 + 2 \bigcirc 5 + 1.4
\]

Add.

\[
4.2 \bigcirc 6.4
\]

\[
4.2 \lessdot 6.4, \text{ so } \sqrt{5} + 2 \lessdot 5 + \sqrt{2}.
\]

Problem 2

Order $\sqrt{7}$, $\pi - 1$, and 2.5 from least to greatest.

Find approximate values for $\sqrt{7}$ and $\pi - 1$.

\[
\sqrt{7} \approx 2.65 \quad \pi - 1 \approx 3.14 - 1
\]

\[
\approx 2.14
\]

Plot the three values on a number line.

From least to greatest, the numbers are $\pi - 1$, 2.5, and $\sqrt{7}$

1. Compare. Write $<$, $>$, or $=$.

\[
\sqrt{13} + 8 \bigcirc \sqrt{8} + 13
\]

2. Order $\sqrt{19}$, $\pi + 1$, and 4.4 from least to greatest. ______________

3. Name a situation in which it would be very important to know the order of a series of numbers.

_______________________________________________________________________________________
Real Numbers

Challenge

Venn Diagrams

Diagrams using circles can show relationships between classes or sets. These diagrams are named after the English mathematician John Venn who introduced them.

Here are some examples using integers, even numbers, odd numbers, and primes. Remember that just one prime number, 2, is even. A shaded region is empty. A region marked \( x \) has at least one member.

\[ O \cap I \]

All odd numbers are integers.

\[ O \cap E \]

No odd numbers are even.

\[ E \cap P \]

Some even numbers are prime.

All \( O \) is \( I \).

No \( O \) is \( E \).

Some \( E \) is \( P \).

Describe each diagram using both words and letters. Set \( S \) is the square numbers.

1. 
2. 
3. 

Draw a Venn diagram for each statement.

4. Some square numbers are odd.
5. No prime numbers are squares.

6. All square numbers are integers.
7. Some odd numbers are not prime.
Find the value of each power.

1. \(5^3 = \) ____________  
2. \(7^{-2} = \) ____________  
3. \(51^1 = \) ____________  
4. \(3^{-4} = \) ____________  
5. \(1^{12} = \) ____________  
6. \(64^0 = \) ____________  
7. \(4^{-3} = \) ____________  
8. \(4^3 = \) ____________  
9. \(10^5 = \) ____________

Find the missing exponent.

10. \(n^3 = n \quad \cdot \quad n^{-3} \)  
11. \(\frac{a}{a^2} = a^4 \)
12. \((r^4) = r^{12} \)

Simplify each expression.

13. \((9 - 3)^2 - (5 \cdot 4)^0 = \) ____________  
14. \((2 + 3)^5 \div (5^2)^2 = \) ____________
15. \(4^2 \div (6 - 2)^4 = \) ____________  
16. \([(1 + 7)^2]^2 \cdot (12^2)^0 = \) ____________

Use the description below to complete Exercises 17–20.

A shipping company makes a display to show how many cubes can fit into a large box. Each cube has sides of 2 inches. The large box has sides of 10 inches.
17. Use exponents to express the volume of each cube and the large box.

Volume of cube = ____________  
Volume of large box = ____________

18. Find how many cubes will fit in the box. ________________

19. Suppose the shipping company were packing balls with a diameter of 2 inches instead of cubes. Would the large box hold more balls or fewer balls than boxes? Explain your answer.

______________________________________________________________  
______________________________________________________________  
______________________________________________________________

20. Suppose the size of each cube is doubled and the size of the large box is doubled. How many of these new cubes will fit in that new large box? Explain how you found your answer.

______________________________________________________________  
______________________________________________________________  
______________________________________________________________

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LESSON 2-1
Integer Exponents
Practice and Problem Solving: C

Simplify each expression.

1. \((7 - 3)^2 \cdot (6 - 2)^3 = \) ____________
2. \((7 - 3)^2 \div (6 - 2)^3 = \) ____________
3. \((2 \cdot 5^3) \div (9 - 4)^4 = \) ____________
4. \([2 + 7]^2 \cdot (10^2)^0 = \) ____________
5. \((3 \cdot 4)^2 \div (6 \cdot 2)^4 = \) ____________
6. \([2^2]^2 \cdot 2^3 = \) ____________

Answer each question.

7. Andrea quickly gave the answer to the problem below. Can you do the same? Explain how you found your answer.

   Find the value of \(a^n \cdot a^{n-1} \cdot a^{n-2} \cdot a^{n-3} \cdot a^{n-4} \cdot a^{n-5} \cdot a^{n-6}\)
   when \(a = 2\) and \(n = 3\).

   __________________________________________________________
   __________________________________________________________

For each experiment, make a prediction first. Then complete the given table. Finally, try the experiment and see if your prediction is correct.

   Experiment 1: Fold a piece of paper in half over and over again to make smaller and smaller rectangles.

8. Predict the maximum number of small rectangles you can make before you cannot fold the paper any further. ________________

9. ______

<table>
<thead>
<tr>
<th>Number of Folds</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Rectangles</td>
<td>(2^0), 1</td>
<td>(2^1), 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. Do the experiment. How many rectangles could you make? ________________

   Experiment 2: Cut a piece of paper in half. Make a single pile of the pieces. Cut the pile in half. Continue making a single pile of the pieces and cutting the pile in half.

11. Predict the maximum number of pieces you can make before you cannot cut the paper any further. ________________

12. Do the experiment. How many pieces could you make? ________________
LESSON 2-1 Integer Exponents

Practice and Problem Solving: D

Write each expression without exponents. Then find the value. The first one is done for you.

1. \(4^{-4} = \frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}\)

2. \(6^2 = \) ___________  

3. \(3^5 = \) ___________  

4. \(24^0 = \) ___________  

5. \(7^{-2} = \) ___________  

6. \(10^5 = \) ___________  

Simplify each expression. Show your work. The first is done for you.

7. \(\frac{(3 \cdot 2)^6}{(7 - 1)^4} = \frac{6^6}{6^4} = 6^{6 - 4} = 6^2\)

8. \((3^2 \cdot 3^1) = 9 \cdot 3 = 27\)

9. \(4^2 \cdot 4^3 = 16 \cdot 64 = 1024\)

10. \((4^2)^3 = 16^3 = 4096\)

11. \((4 - 3)^2 \cdot (5 \cdot 4)^0 = 1 \cdot 1 = 1\)

12. \((2 + 3)^5 \div (5^2)^2 = 5^5 \div 25^2 = 5^3 = 125\)

Answer the question.

13. Find the value of \((2^2)^3\). Then find the value of \((2^3)^2\). What is true about the results? Explain why.

_________________________________________________________________________________________

_________________________________________________________________________________________

_________________________________________________________________________________________
A positive exponent tells you how many times to multiply the base as a factor. A negative exponent tells you how many times to divide by the base. Any number to the 0 power is equal to 1.

\[ 4^2 = 4 \cdot 4 = 16 \quad 4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024 \quad a^3 = a \cdot a \cdot a \]

\[ 4^{-2} = \frac{1}{4^2} = \frac{1}{4 \cdot 4} = \frac{1}{16} \quad 4^{-5} = \frac{1}{4^5} = \frac{1}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{1024} \quad a^{-3} = \frac{1}{a^3} = \frac{1}{a \cdot a \cdot a} \]

When you work with integers, certain properties are always true. With integer exponents, there are also certain properties that are always true.

When the bases are the same and you multiply, you add exponents.

\[ 2^2 \cdot 2^4 = 2^2+4 \quad a^m \cdot a^n = a^{m+n} \]

\[ 2\cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 \]

When the bases are the same and you divide, you subtract exponents.

\[ \frac{2^5}{2^3} = 2^{5-3} \quad \frac{a^m}{a^n} = a^{m-n} \]

\[ \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2^2 \]

When you raise a power to a power, you multiply.

\[ (2^3)^2 = 2^3 \cdot 2 \quad (a^m)^n = a^{m \cdot n} \]

\[ (2 \cdot 2 \cdot 2)^2 \quad (2 \cdot 2 \cdot 2) = 2^6 \]

Tell whether you will add, subtract, or multiply the exponents. Then simplify by finding the value of the expression.

1. \[ \frac{3^6}{3^3} \rightarrow \] 2. \[ 8^2 \cdot 8^{-3} \rightarrow \]
3. \[ (3^2)^3 \rightarrow \] 4. \[ 5^3 \cdot 5^1 \rightarrow \]
5. \[ \frac{4^2}{4^4} \rightarrow \] 6. \[ (6^3)^2 \rightarrow \]
LESSON 2-1  Integer Exponents

Reading Strategies: Using Patterns

You can use patterns to help evaluate powers.

Look at the patterns in each column. As you move down the column, you will note that the products are getting smaller. That is because there is one less factor when the powers are positive and one more factor when the powers are negative.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^3 = 8$</td>
<td>$3^3 = 27$</td>
<td>$4^3 = 64$</td>
</tr>
<tr>
<td>$2^2 = 4$</td>
<td>$3^2 = 9$</td>
<td>$4^2 = 16$</td>
</tr>
<tr>
<td>$2^1 = 2$</td>
<td>$3^1 = 3$</td>
<td>$4^1 = 4$</td>
</tr>
<tr>
<td>$2^0 = 1$</td>
<td>$3^0 = 1$</td>
<td>$4^0 = 1$</td>
</tr>
<tr>
<td>$2^{-1} = \frac{1}{2}$</td>
<td>$3^{-1} = \frac{1}{3}$</td>
<td>$4^{-1} = \frac{1}{4}$</td>
</tr>
<tr>
<td>$2^{-2} = \frac{1}{4}$</td>
<td>$3^{-2} = \frac{1}{9}$</td>
<td>$4^{-2} = \frac{1}{16}$</td>
</tr>
</tbody>
</table>

Use the table to answer each question.

1. Describe the pattern of the exponents in each column.

2. What is the base of column 2?

3. In column 2, what is the product divided by each time to get the product in the cell below?

4. What is the base of column 3?

5. In column 3, what is the product divided by each time to get the product in the cell below?

Complete the table, using the table above as a guide.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^3 = 125$</td>
<td>$6^3 = 216$</td>
<td>$10^3 = 1000$</td>
</tr>
<tr>
<td>$5^2 = 25$</td>
<td>$6^2 = 36$</td>
<td>$10^2 = 100$</td>
</tr>
</tbody>
</table>
The set of integers is the set of whole numbers and their opposites, such as \(3, 2, 1, 0, -1, -2,\) and \(-3\). Integer exponents are powers of a number where the power is a whole number or its opposite.

**Problem 1**

\[
4^2 = 4 \cdot 4 = 16 \\
4^{-2} = \frac{1}{4^2} = \frac{1}{4 \cdot 4} = \frac{1}{16} \\
4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024 \\
4^{-5} = \frac{1}{4^5} = \frac{1}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{1024} \\
a^3 = a \cdot a \cdot a \\
a^{-3} = \frac{1}{a^3} = \frac{1}{a \cdot a \cdot a}
\]

**Problem 2**

When you work with integers, certain properties are always true. With integer exponents, there are also certain properties that are always true.

Use properties of exponents to simplify each expression.

\[
\frac{2^4}{2^2} = 2^{4-2} = 2^2 \\
\frac{2^5}{2^4} = 2^{5-4} = 2^1 = 2 \\
\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2 \\
\frac{(2^3)^2}{2^2} = 2^{3 \cdot 2} = 2^6 \\
(2 \cdot 2 \cdot 2)^2 = 2^6 \\
2 \cdot 2 \cdot 2 \cdot 2 = 2^4
\]

Complete.

1. Explain in your own words what a negative exponent means.

   ____________________________________________________________________________

   ____________________________________________________________________________

Use properties of exponents to simplify each expression.

2. \(\frac{3^6}{3^4} = \) ____________  
3. \(4^2 \cdot 4^1 = \) ____________  
4. \((x^5)^4 = \) ____________

5. \((4^2)^3 = \) ____________  
6. \(12^3 \cdot 12^{-2} = \) ____________  
7. \(z^6 \cdot z^6 = \) ____________
Write each number as a power of 10.
1. 100
2. 10,000
3. 100,000
4. 10,000,000
5. 1,000,000
6. 1000
7. 1,000,000,000
8. 1

Write each power of ten in standard notation.
9. \(10^3\)
10. \(10^5\)
11. 10
12. \(10^6\)
13. \(10^2\)
14. \(10^0\)
15. \(10^4\)
16. \(10^7\)

Write each number in scientific notation.
17. 2500
18. 300
19. 47,300
20. 24
21. 14,565
22. 7001
23. 19,050,000
24. 33

Write each number in standard notation.
25. \(6 \times 10^3\)
26. \(4.5 \times 10^2\)
27. \(7 \times 10^7\)
28. \(1.05 \times 10^4\)
29. \(3.052 \times 10^3\)
30. \(5 \times 10^0\)
31. \(9.87 \times 10^1\)
32. \(5.43 \times 10^1\)

Solve.
33. The average distance of the Moon from Earth is about 384,400 kilometers. Write this number in scientific notation.

34. The radius of Earth is about 6.38 \(\times 10^3\) kilometers. Write this distance in standard notation.
LEsson 2-2 Scientific Notation with Positive Powers of 10

Practice and Problem Solving: C

Write each pair of numbers in standard notation. Use the symbols >, <, or = to compare them. Show your work.

1. $2.5 \times 10^3$ ___ $2.5 \times 10^6$

2. $5 \times 10^6$ ___ $2.5 \times 10^6$

3. $3 \times 10^0$ ___ $1 \times 10^1$

4. $4.025 \times 10^3$ ___ $1.025 \times 10^4$

Write each pair of numbers in scientific notation. Write the numbers in scientific notation on the correct side of the comparison symbol.

5. 1200; 450

6. 230,000; 32,000

Write the numbers from least to greatest.

7. $3.25 \times 10^5$, $5.32 \times 10^5$, $2.35 \times 10^6$, $5.32 \times 10^6$, $3.25 \times 10^5$, $2.35 \times 10^5$

8. $5 \times 10^0$, $1 \times 10^1$, $0 \times 10^0$, $1 \times 10^0$, $5 \times 10^1$

Use the fact that 1 meter equals $10^3$ millimeters and 1 centimeter equals $10^1$ millimeters to label each of these statements as true or false. Show your work.

9. $1 \times 10^3$ m ___ $1 \times 10^5$ cm

10. $9 \times 10^1$ m ___ $9 \times 10^1$ mm

True or false?

Solve.

11. An athletic stadium has a capacity of $1.5 \times 10^4$ fans. If $9 \times 10^3$ fans buy advance tickets to an event at the stadium, how many tickets will be available at the box office on the day of the event? Show your work by writing the numbers in standard notation.

12. A town’s most popular drive-in restaurant has 2,500 followers on a social networking website. The town’s high school athletic program has $1.5 \times 10^4$ followers on the same website. Which is more popular according to this data, the drive-in or the athletic program? Show your work by writing the numbers in standard notation.
**Scientific Notation with Positive Powers of 10**

**Practice and Problem Solving: D**

**Write each product in standard form. The first one is done for you.**

1. \( 10 \times 10 = \underline{100} \)
2. \( 10 \times 10 \times 10 \times 10 \times 10 = \)
3. \( 10 \times 10 \times 10 \times 10 = \)
4. \( 10 \times 10 \times 10 = \)

**Write each number as a product of tens. The first one is done for you.**

5. \( 100,000 = 10 \times 10 \times 10 \times 10 \times 10 \)
6. \( 10,000,000 = \)
7. \( 10,000 = \)
8. \( 100,000,000,000 = \)

**Write each number as a power of ten and an exponent. The first one is done for you.**

9. \( 1000 = 10^3 \)
10. \( 10 = \) \( 10^1 \)
11. \( 100,000 = \)

**Write each power of ten in standard form. The first one is done for you.**

12. \( 10^1 \)
13. \( 10^3 \)
14. \( 10^4 \)
15. \( 10^0 \)
16. \( 10^5 \)
17. \( 10^0 \)

**Write the exponent for the question mark. The first one is done for you.**

18. \( 3600 = 3.6 \times 10^2 \) \( 3 \)
19. \( 450 = 4.5 \times 10^2 \) \( \) \( \) \( \)
20. \( 5,000,000 = 5 \times 10^7 \) \( \) \( \) \( \)
21. \( 6 = 6 \times 10^7 \) \( \) \( \) \( \)

**Write each number in standard form. The first one is done for you.**

22. \( 3.56 \times 10^3 = \underline{3,560} \)
23. \( 9 \times 10^3 = \)
24. \( 6.875 \times 10^4 = \)
25. \( 4.005 \times 10^6 = \)

**Solve.**

26. The volume of a cube is 10 feet times 10 feet times 10 feet. Write this product as one number in standard form.
**LESSON 2-2**

**Scientific Notation with Positive Powers of 10**

**Reteach**

You can change a number from standard notation to scientific notation in 3 steps.
1. Place the decimal point between the first and second digits on the left to make a number between 1 and 10.
2. Count from the decimal point to the right of the last digit on the right.
3. Use the number of places counted in Step 2 as the power of ten.

**Example**

Write 125,000 in scientific notation.

\[
\begin{align*}
1.25 & \quad 1) \text{ The first and second digits to the left are 1 and 2, so place the decimal point between the two digits to make the number 1.25.} \\
125,000 & \quad 2) \text{ The last digit in 125,000 is 5 places to the right.} \\
1.25 \times 10^5 & \quad 3) \text{ The power of 10 is 5.}
\end{align*}
\]

You can change a number from scientific notation to standard notation in 3 steps.
1. Find the power of 10.
2. Count that number of places to the right.
3. Add zeros as needed.

**Example**

Write \(5.96 \times 10^4\) in standard notation.

\[
\begin{align*}
10^4 & \quad 1) \text{ The power of 10 is 4.} \\
5.9600 & \quad 2) \text{ Move the decimal point 4 places to the right.} \\
59,600 & \quad 3) \text{ Add two zeros.}
\end{align*}
\]

Complete to write each number in scientific notation.

1. 34,600
   - The number between 1 and 10: ____
   - The power of 10: ____
   - The number in scientific notation: _____________

2. 1,050,200
   - The number between 1 and 10: ____
   - The power of 10: ____
   - The number in scientific notation: _____________

Write each number in standard notation.

3. \(1.057 \times 10^3\)
4. \(3 \times 10^8\)
5. \(5.24 \times 10^5\)

_______________________
_______________________
_______________________
**Scientific Notation with Positive Powers of 10**

*Reading Strategies: Follow a List of Steps*

Lists can help you understand the steps of changing from standard notation to scientific notation.

**Change a number from standard notation to scientific notation**

A. Locate the digit that is on the left end of the number. For example, if the number is 2350, the digit on the far left is “2.”

B. Place the decimal point *after or to the right* of that digit. For example, in the example 2350, moving the decimal makes the number “2.350”. This gives a number between 1 and 10.

C. Count places to the right from the new decimal point to the end of the number. How many is that? In the example, it is *three places* from the new decimal point to the end of the number.

D. Use the number of places as the *exponent* or *power* of ten after you write the number between 1 and 10, a times sign, and a “10.” In this example, the “3” goes with the “10.” This gives $2.350 \times 10^3$. After moving the decimal, zeros at the end of the number can be dropped, so $10^3$ can be written as $2.35 \times 10^3$.

Fill in the steps in the list below each number as you write it in scientific notation.

1. 295  
2. 10,500  
3. 4,505,000

A: ____________________  
B: ____________________  
C: ____________________  
D: ____________________

**Change a number from scientific notation to standard notation**

A. Find the decimal point in the number between 1 and 10 in a number written in scientific notation. For example, in $4.56 \times 10^7$ the decimal is between the digits “4” and “5”.

B. Locate the exponent or power of 10. In the example $4.56 \times 10^7$ the power of 10 is “7.”

C. Starting at the decimal, count seven places to the right. If there are no numbers to be counted, write zeros. In the example $4.56 \times 10^7$, you would count the “5” and the “6” as two places, and add five zeros. This would give 45600000, or 45,600,000.

Fill in the steps below each number as you write it in standard notation.

4. $2.5 \times 10^4$  
5. $7 \times 10^5$  
6. $1.234 \times 10^3$

A: ____________________  
B: ____________________  
C: ____________________
Problem 1
Write $3.12 \times 10^9$ in standard notation.
Which way should you move the decimal point?

Think about the number line.

The exponent 9 is positive, so the decimal point moves to the right.
So, $3.12 \times 10^9$ in standard notation is 3,120,000,000.

Problem 2
Write 7,505,000 in scientific notation.

7,505,000 $\leftarrow$ Move decimal point six places to the left.

When you move the decimal to the left, the exponent of 10 is positive.
So, 7,505,000 in scientific notation is $7.505 \times 10^6$.

1. Which number is greater, $3.28 \times 10^5$ or $3.28 \times 10^3$? Prove your answer by writing each number in standard notation.

2. Is $3 \times 10^6$ grams more likely to be the mass of a car or the mass of an eyelash? Explain.

3. Write 186,000 in scientific notation.

4. Write $4.56789 \times 10^3$ in standard form.
Scientific Notation with Negative Powers of 10

Practice and Problem Solving: A/B

Write each number as a negative power of ten.
1. \( \frac{1}{10^2} = \) _______
2. \( \frac{1}{10^4} = \) _______
3. \( \frac{1}{10^5} = \) _______
4. \( \frac{1}{10^7} = \) _______
5. \( \frac{1}{10^6} = \) _______
6. \( \frac{1}{10^3} = \) _______
7. \( \frac{1}{10^9} = \) _______
8. \( \frac{1}{10^1} = \) _______

Write each power of ten in standard notation.
9. \( 10^{-3} = \) _______
10. \( 10^{-5} = \) _______
11. \( 10^{-1} = \) _______
12. \( 10^{-6} = \) _______
13. \( 10^{-2} = \) _______
14. \( 10^{-9} = \) _______
15. \( 10^{-4} = \) _______
16. \( 10^{-7} = \) _______

Write each number in scientific notation.
17. 0.025
18. 0.3
19. 0.000473
20. 0.0024
21. 0.000014565
22. 0.70010
23. 0.0190500
24. 0.00330000

Write each number in standard notation.
25. \( 6 \times 10^{-3} \)
26. \( 4.5 \times 10^{-2} \)
27. \( 7 \times 10^{-7} \)
28. \( 1.05 \times 10^{-6} \)
29. \( 3.052 \times 10^{-8} \)
30. \( 5 \times 10^{-1} \)
31. \( 9.87 \times 10^{-4} \)
32. \( 5.43 \times 10^{-5} \)

Solve.
33. An \( E. coli \) bacterium has a diameter of about \( 5 \times 10^{-7} \) meter. Write this measurement as a decimal in standard notation.

34. A human hair has an average diameter of about 0.000017 meter. Write this measurement in scientific notation.
Write each pair of numbers in standard notation. Use the symbols >, <, or = to compare them.

1. $5.2 \times 10^{-3}$ ☐ $5.2 \times 10^{-6}$
2. $5 \times 10^{-6}$ ☐ $2.5 \times 10^{-5}$
3. $3 \times 10^{0}$ ☐ $1 \times 10^{-1}$
4. $5.02 \times 10^{-3}$ ☐ $2.05 \times 10^{-4}$

Write each pair of numbers in scientific notation. Write the numbers in scientific notation on the correct side of the comparison symbol.

5. 0.0012; 0.45
6. 0.0000023; 0.00032

List the numbers in order from least to greatest.

7. $3.25 \times 10^{-6}$, $5.32 \times 10^{-5}$, $2.35 \times 10^{-6}$, $5.32 \times 10^{-6}$, $3.25 \times 10^{-5}$, $2.35 \times 10^{-5}$

8. $5 \times 10^{0}$, $1 \times 10^{-1}$, $0 \times 10^{0}$, $1 \times 10^{0}$, $5 \times 10^{-1}$

Identify whether each statement is true or false. Circle the correct answer. Show your work. (1 m = $10^3$ mm; 1 cm = $10^1$ mm)

9. $1 \times 10^{-3}$ m ☐ $1 \times 10^{-1}$ cm
True or false?

10. $7 \times 10^{-1}$ cm ☐ $7 \times 10^{-3}$ m
True or false?

11. $3.5 \times 10^{-1}$ cm ☐ $3.5 \times 10^{-3}$ m
True or false?

12. $9 \times 10^{-1}$ mm ☐ $9 \times 10^{-4}$ m
True or false?

Solve.

13. A test tube used in science class has a volume capacity of $9 \times 10^{-3}$ liter. How many drops of a solution will it take to fill the test tube if each drop’s volume is $3 \times 10^{-5}$ liter? Write these numbers in standard notation and then calculate the answer.

14. A logic array on a semiconductor chip has a rectangular shape. Its length is 0.00025 meter and its width is 0.000125 meter. What is the logic array’s area? Write these numbers in scientific notation.
Scientific Notation with Negative Powers of 10

**Practice and Problem Solving: D**

Write each product in standard form. The first one is done for you.

1. \( \frac{1}{10 \times 10} = \frac{1}{100} \)
2. \( \frac{1}{10 \times 10 \times 10 \times 10 \times 10} = \) 
3. \( \frac{1}{10 \times 10 \times 10 \times 10} = \)
4. \( \frac{1}{10 \times 10} = \)

Write each number as a product of tens. The first one is done for you.

5. \( \frac{1}{100,000} = \frac{1}{10 \times 10 \times 10 \times 10 \times 10} \)
6. \( \frac{1}{10,000,000} = \)
7. \( \frac{1}{10,000} = \)
8. \( \frac{1}{100,000,000,000} = \)

Write each number as both a power of ten and a negative exponent. The first one is done for you.

9. \( \frac{1}{1000} = \frac{1}{10^3} = 10^{-3} \)
10. \( \frac{1}{10} = \)
11. \( \frac{1}{100} = \)
12. \( \frac{1}{10,000} = \)

Write in standard form. The first one is done for you.

13. \( \frac{1}{10} = \)
14. \( \frac{1}{10^3} = \)
15. \( \frac{1}{10^4} = \)
16. \( \frac{1}{10^9} = \)
17. \( \frac{1}{10^5} = \)
18. \( \frac{1}{10^{12}} = \)

Identify the unknown exponent. The first one is done for you.

19. \( 0.00036 = 3.6 \times 10^{-4} \)
20. \( 0.450 = 4.5 \times 10^{-7} \)
21. \( 0.000000005 = 5 \times 10^{-7} \)
22. \( 0.00600 = 6 \times 10^{-7} \)

Write each number in standard form. The first one is done for you.

23. \( 3.56 \times 10^{-3} = 0.00356 \)
24. \( 9 \times 10^{-5} = \)
25. \( 6.875 \times 10^{-4} = \)
26. \( 4.005 \times 10^{-6} = \)

Solve.

27. The volume of a box is found by multiplying its length, width, and height. The three sides are 0.5 foot, 0.75 foot, and 0.4 foot. Find the product. Write it in scientific notation.
Scientific Notation with Negative Powers of 10

Reteach

You can convert a number from standard form to scientific notation in 3 steps.
1. Starting from the left, find the first non-zero digit. To the right of this digit is the new location of your decimal point.
2. Count the number of places you moved the decimal point. This number will be used in the exponent in the power of ten.
3. Since the original decimal value was less than 1, your power of ten must be negative. Place a negative sign in front of the exponent.

Example
Write 0.00496 in standard notation.

4.96 1) The first non-zero digit is 4, so move the decimal point to the right of the 4.
4.96 × 10³ 2) The decimal point moved 3 places, so the whole number in the power of ten is 3.
4.96 × 10⁻³ 3) Since 0.00496 is less than 1, the power of ten must be negative.

You can convert a number from scientific notation to standard form in 3 steps.
1. Find the power of ten.
2. If the exponent is negative, you must move the decimal point to the left. Move it the number of places indicated by the whole number in the exponent.
3. Insert a leading zero before the decimal point.

Example
Write 1.23 × 10⁻⁵ in standard notation.

10⁻⁵ 1) Find the power of ten.
.0000123 2) The exponent is −5, so move the decimal point 5 places to the left.
0.0000123 3) Insert a leading zero before the decimal point.

Write each number in scientific notation.
1. 0.0279 2. 0.00007100 3. 0.0000005060

Write each number in standard notation.
4. 2.350 × 10⁻⁴ 5. 6.5 × 10⁻³ 6. 7.07 × 10⁻⁵
Scientific Notation with Negative Powers of 10

Reading Strategies: Use Graphic Aids

Change a number from standard notation to scientific notation

Change 0.0000003 to scientific notation.

A. What is the first non-zero digit? It is 3.
B. Move the decimal point until it is directly to the right of the 3. The decimal point moves 7 places to the right.
C. The decimal point moved 7 places to the right. So, the power of ten is 7, and it must be negative. The power of ten is \(-7\).
D. So, the number in scientific notation is \(3.0 \times 10^{-7}\)

Fill in the steps below each number to write it in scientific notation.

1. 0.00123
   First non-zero digit: ____________
   Number of places from decimal: ____
   Direction decimal point moves: _____
   Power of 10: ________________
   Scientific notation: ________________

2. 0.000567
   First non-zero digit: ____________
   Number of places from decimal: ____
   Direction decimal point moves: _____
   Power of 10: ________________
   Scientific notation: ________________

Change a number from scientific notation to standard notation

Change \(2.5 \times 10^{-4}\) to standard notation.

A. What is the power of 10? It is \(-4\).
B. Since the power of 10 is negative, the decimal must move left.
   Move the decimal point 4 places to the left.
C. So, the number in standard notation is 0.00025.

Fill in the steps below each number to write it in standard notation.

3. \(6.7 \times 10^{-8}\)
   Power of 10: ________________
   Direction decimal point moves: _____
   Number of places: ____________
   Standard notation: ________________

4. \(3.21 \times 10^{-4}\)
   Power of 10: ________________
   Direction decimal point moves: _____
   Number of places: ____________
   Standard notation: ________________
Problem 1
Write $5.43 \times 10^{-6}$ in standard notation.
Which way should you move the decimal point?

Think about the number line.

The exponent ($-6$) is negative, so the decimal point moves to the left.
So, $5.43 \times 10^{-6}$ in standard notation is $0.00000543$.

Problem 2
Write $0.00000456$ in scientific notation.

Move decimal point six places to the right.

When you move the decimal to the right, the exponent is negative.
So, $0.00000456$ in scientific notation is $4.56 \times 10^{-6}$.

1. Which number is greater, $5.75 \times 10^{-3}$ or $5.75 \times 10^{-4}$? Prove your answer by writing each number in standard notation.

_________________________________________________________________________________________

2. Is $3 \times 10^{-7}$ grams more likely to be the mass of a bicycle or the mass of a hair? Explain.

_________________________________________________________________________________________

_________________________________________________________________________________________

3. Write $0.000493$ in scientific notation.

4. Write $3.21 \times 10^{-5}$ in standard form.

________________________________________  _____________________________________
LESSON 2-4
Operations with Scientific Notation

Practice and Problem Solving: A/B

Add or subtract. Write your answer in scientific notation.

1. \(6.4 \times 10^3 + 1.4 \times 10^4 + 7.5 \times 10^3\)

2. \(4.2 \times 10^6 - 1.2 \times 10^5 - 2.5 \times 10^5\)

3. \(3.3 \times 10^9 + 2.6 \times 10^9 + 7.7 \times 10^8\)

4. \(8.0 \times 10^4 - 3.4 \times 10^4 - 1.2 \times 10^3\)

Multiply or divide. Write your answer in scientific notation.

5. \((3.2 \times 10^8)(1.3 \times 10^6) = \frac{8.8 \times 10^7}{4.4 \times 10^4}\)

6. \(\frac{8.8 \times 10^7}{4.4 \times 10^4} = \frac{1.44 \times 10^{10}}{2.4 \times 10^2}\)

Write each number using calculator notation.

9. \(4.1 \times 10^4 = \frac{8.8 \times 10^7}{4.4 \times 10^4}\)

10. \(9.4 \times 10^{-6} = \frac{1.44 \times 10^{10}}{2.4 \times 10^2}\)

Write each number using scientific notation.

11. \(5.2E-6 = \frac{8.8 \times 10^7}{4.4 \times 10^4}\)

12. \(8.3E+2 = \frac{1.44 \times 10^{10}}{2.4 \times 10^2}\)

Use the situation below to complete Exercises 13–16. Express each answer in scientific notation.

A runner tries to keep a consistent stride length in a marathon. But, the length will change during the race. A runner has a stride length of 5 feet for the first half of the race and a stride length of 4.5 feet for the second half.

13. A marathon is 26 miles 385 yards long. That is about \(1.4 \times 10^6\) feet. How many feet long is half a marathon?

14. How many strides would it take to finish the first half of the marathon?

15. How many strides would it take to finish the second half of the marathon?

16. How many strides would it take the runner to complete marathon? Express your answer in both scientific notation and standard notation.

Hint: Write 5 ft as \(5.0 \times 10^0\) and 4.5 feet as \(4.5 \times 10^0\).
Operations with Scientific Notation

Practice and Problem Solving: C

Add or subtract. Write your answer in scientific notation.

1. \(2.4 \times 10^2 + 3.4 \times 10^4 + 1.5 \times 10^3\)

2. \(6.2 \times 10^4 - 3.4 \times 10^2 - 7.5 \times 10^3\)

3. \(8.3 \times 10^5 + 1.6 \times 10^7 + 6.7 \times 10^4\)

4. \(8.0 \times 10^3 - 0.8 \times 10^3 - 1.2 \times 10^2\)

Multiply or divide. Write your answer in scientific notation.

5. \((5.2 \times 10^8)(4.8 \times 10^4) =\)

6. \(\frac{9.8 \times 10^7}{1.4 \times 10^{-5}} =\)

7. \((8.5 \times 10^2)(3.4 \times 10^{-5}) =\)

8. \(\frac{1.702 \times 10^5}{7.4 \times 10^3} =\)

Use the information below to complete Exercises 9–13.

A 60-watt light bulb uses 60 watt hours of electricity in 1 hour. Suppose everyone in the United States left one unneeded 60 watt light bulb on for one hour every day for a year.

9. Electricity is billed in kilowatt hours. So 60 watt hours is equal to sixty divided by one thousand. Express the electricity used by a 60-watt light bulb in one hour in kilowatt hours in scientific notation.

10. Express the number of days in a year in scientific notation.

11. There are about 315,000,000 people in the United States. Write that number in scientific notation.

12. Now find how many kilowatt hours of electricity would be wasted if every person in the United States left one unneeded 60-watt light bulb one hour a day for a whole year. Express your answer in both scientific notation and standard notation.

13. The average household uses about 15,000 kilowatt hours per year. How many households could use that wasted electricity from item 12 and have light for a year? Express your answer in standard notation.
Operations with Scientific Notation

Practice and Problem Solving: D

Add or subtract. Write your answer in scientific notation. The first one is done for you.

1. \(2.4 \times 10^2 + 3.3 \times 10^4 + 7.2 \times 10^3\)
   \[240 + 33,000 + 7200 = 40,440\]
   \[= 4.044 \times 10^4\]

2. \(1.2 \times 10^4 - 1.5 \times 10^3 - 2.2 \times 10^2\)

3. \(7.3 \times 10^5 + 1.6 \times 10^6 + 4.7 \times 10^5\)

Multiply or divide. Write your answer in scientific notation. The first one is done for you.

5. \((3.2 \times 10^3)(6.4 \times 10^6) = (3.2 \times 6.4) \times (10^3 \times 10^6)\)
   \[= 20.48 \times 10^{3+6}\]
   \[= 20.48 \times 10^{12}\]
   \[= 2.048 \times 10^{13}\]

6. \(\frac{9.6 \times 10^5}{5 \times 10^4}\)

7. \((2.5 \times 10^4)(4.1 \times 10^4)\)

Write each number using calculator notation. The first one is done for you.

9. Scientific notation | \(7.1 \times 10^5\) | \(4.4 \times 10^{-3}\)
10. Calculator notation | \(7.1E+5\) | \(3.3E-3\) | \(6.9E+5\)

Answer the questions.

11. How do you write one million in scientific notation? ________________

12. A day is \(8.64 \times 10^4\) seconds long. Write and solve an expression to find how many days are in one million seconds. Give your answer in standard form.

   ________________
Lesso
2-4
Operations with Scientific Notation
Reteach

To add or subtract numbers written in scientific notation:

Check that the exponents of powers of 10 are the same.
If not, adjust the decimal numbers and the exponents.
Add or subtract the decimal numbers.
Write the sum or difference and the common power of 10 in scientific notation format.
Check whether the answer is in scientific notation.
If it is not, adjust the decimal and the exponent.

\[(a \times 10^n) + (b \times 10^m) = (a + b) \times 10^n \quad (1.2 \times 10^5) - (9.5 \times 10^4)\]
\[(a \times 10^n) - (b \times 10^m) = (a - b) \times 10^n \quad (1.2 \times 10^5) - (0.95 \times 10^5)\]
\[\quad \text{← Adjust to get same exponent.} \]
\[0.25 \times 10^5 \quad \text{← Not in scientific notation.} \]
\[2.5 \times 10^4 \quad \text{← Answer} \]

To multiply numbers written in scientific notation:

Multiply the decimal numbers.
Add the exponents in the powers of 10.
Check whether the answer is in scientific notation.
If it is not, adjust the decimal numbers and the exponent.

\[(a \times 10^n) \times (b \times 10^m) = ab \times 10^{n-m} \quad (2.7 \times 10^8) \times (8.9 \times 10^4)\]
\[(2.7 \times 8.9) \times 10^{8-4} \quad 24.03 \times 10^{12} \quad \text{← Not in scientific notation.} \]
\[2.403 \times 10^{13} \quad \text{← Answer} \]

To divide numbers written in scientific notation:

Divide the decimal numbers.
Subtract the exponents in the powers of 10.
Check whether the answer is in scientific notation.
If it is not, adjust the decimal numbers and the exponent.

\[(a \times 10^n) \div (b \times 10^m) = a \div b \times 10^{n-m} \quad (6.3 \times 10^7) \div (9.0 \times 10^3)\]
\[(6.3 \div 9.0) \times 10^{7-3} \quad 0.7 \times 10^4 \quad \text{← Not in scientific notation.} \]
\[7.0 \times 10^3 \quad \text{← Answer} \]

Compute. Write each answer in scientific notation.
1. \((2.21 \times 10^7) \div (3.4 \times 10^4)\)
2. \((5.8 \times 10^6) - (4.3 \times 10^6)\)
3. \((2.8 \times 10^3)(7.5 \times 10^4)\)
A flowchart gives you a plan. You can use a flowchart to compute with numbers given in scientific notation.

To multiply numbers in scientific notation:

1. Multiply the decimal numbers.
2. Add the exponents of 10.
3. Write the product from Step 1 and the power from Step 2 in the format of scientific notation.
4. Check that the decimal is between 1 and 10. If not, adjust the decimal and the exponent so the answer is in scientific notation.

To divide numbers in scientific notation:

1. Divide the decimal numbers.
2. Subtract the exponents of 10.
3. Write the quotient from Step 1 and the power from Step 2 in the format of scientific notation.
4. Check that the decimal is between 1 and 10. If not, adjust the decimal and the exponent so the answer is in scientific notation.

To add or subtract numbers in scientific notation:

1. Check that the exponents on powers of 10 are the same. If not, adjust the decimal and the exponent.
2. As long as the exponents match, add or subtract the decimal numbers.
3. Write the sum or difference from Step 2 and the power from Step 1 in the format of scientific notation.
4. Check that the decimal is between 1 and 10. If not, adjust the decimal and the exponent so the answer is in scientific notation.

Identify which flowchart to follow. Perform the indicated operation.

1. \(4.2 \times 10^3 + 2.4 \times 10^4\)
2. \((1.2 \times 10^4)(1.6 \times 10^6)\)
3. \(\frac{8.8 \times 10^7}{4.4 \times 10^4}\)

4. \((6.4 \times 10^3) ÷ (3.2 \times 10^4)\)
5. \((3.2 \times 10^5) - (1.3 \times 10^7)\)
6. \((7.0 \times 10^6)(4.7 \times 10^3)\)
Operations with Scientific Notation

Success for English Learners

When completing mathematical operations with integers, there are procedures that simplify the process.

Problem 1
You can add or subtract numbers in scientific notation.

\[
(5.9 \times 10^5) - (5.4 \times 10^4) \quad \text{← Check to see if each power of 10 has the same exponent.}
\]

\[
(5.9 \times 10^5) - (0.54 \times 10^5) \quad \text{← Adjust to get the same exponent.}
\]

\[
(5.9 - 0.54) \times 10^5 \quad \text{← Add or subtract the decimal parts.}
\]

\[
5.36 \times 10^5 \quad \text{← Use 10 to the common power.}
\]

\[
5.36 \times 10^5 \quad \text{← Check that the answer is in scientific notation.}
\]

Problem 2
You can multiply or divide numbers in scientific notation.

\[
(3.5 \times 10^5) \times (9.1 \times 10^4) \quad \text{← Regroup so decimals and powers of 10 are grouped.}
\]

\[
(3.5 \times 9.1) \times (10^5 \times 10^4) \quad \text{← Multiply or divide the decimal parts.}
\]

\[
(3.5 \times 9.1) \times 10^{5+4} \quad \text{If multiplying, add exponents. If dividing, subtract exponents.}
\]

\[
31.85 \times 10^{12} \quad \text{← Check that the answer is in scientific notation.}
\]

\[
3.185 \times 10^{13} \quad \text{← Answer in scientific notation.}
\]

Complete.

1. When is scientific notation most useful?

________________________________________________________________________________________
________________________________________________________________________________________

Compute. Write each answer in scientific notation.

2. \[
(1.43 \times 10^5) \div (2.6 \times 10^2)
\]

3. \[
(2.8 \times 10^6) - (1.3 \times 10^5)
\]

4. \[
(5.8 \times 10^6)(2.5 \times 10^3)
\]

5. \[
(2.9 \times 10^5) + (8.4 \times 10^4)
\]

6. \[
(1.8 \times 10^3)(3.4 \times 10^6)
\]

7. \[
(3.024 \times 10^9) \div (5.4 \times 10^6)
\]

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Exponents and Scientific Notation

Astronomical Distances

1. A light year is the distance light travels in a vacuum in one year. It is about $9.4605284 \times 10^{15}$ meters. Write this in standard form.

2. Change a light year to kilometers. Write it in both standard form and scientific notation. About how many trillions is this number?

3. Distances to stars are often measured in parsecs rather than in light years. One parsec is about 3.26 light years. Use scientific notation to write the number of kilometers in one parsec to three significant digits.

Subatomic Distances

4. Dimensions of subatomic particles are often described in nanometers. A nanometer is one-billionth of a meter. Write a nanometer in both standard form and scientific notation.

5. Sometimes atomic scale dimensions are written in angstroms. An angstrom equals 0.1 nanometer. Use scientific notation to show the value of an angstrom in meters.

6. A nanometer equals $\frac{1}{1000}$ of a micron. Use scientific notation to show the value of a micron in meters.

7. How many times larger than a nanometer is a parsec?
UNIT 1: Real Numbers, Exponents, and Scientific Notation

MODULE 1 Real Numbers

LESSON 1-1

Practice and Problem Solving: A/B

1. 0.125
2. 0.5625
3. 0.55
4. 5.32
5. 0.93
6. 2.583
7. 0.03
8. 3.2
9. 5, −5
10. 1, −1
11. \(\frac{5}{2}, -\frac{5}{2}\)
12. \(\frac{11}{7}, -\frac{11}{7}\)
13. 2
14. 6
15. 1
16. 13
17. 5.66
18. 10.86
19. 4.24
20. 17.86
21. 2.83
22. 8.66
23. \(\frac{23}{100}\) lb
24. 288 in²

Practice and Problem Solving: C

1. 1.17 in.
2. 7.64 in.
3. Since \(\frac{41}{50} = 0.82, \frac{9}{11} = 0.81\), and \(\frac{10}{11} = 0.90\), \(\frac{41}{50}\) is closer to \(\frac{9}{11}\).
4. ± \(\frac{1}{2}\)
5. ± \(\frac{1}{4}\)
6. ± \(\frac{2}{3}\)
7. ± 1.73
8. ± 4.24

Practice and Problem Solving: D

1. 0.5
2. 0.55
3. 0.5625
4. \(\frac{129}{500}\)
5. \(\frac{44}{5}\)
6. \(\frac{333}{1000}\)
7. 4, −4
8. 7, −7

9. When you find a square root, you find two factors that are the same. They can be positive or negative. When you find a cube root, you find three factors that are the same. They are positive.

10. The length of one photo album page is 25 cm. The width of the page is 32 cm.
9. \( \frac{5}{2}, -\frac{5}{2} \)
10. 7
11. 1
12. \( \frac{2}{3} \)
13. 5.66
14. 7.68
15. 10.86
16. 0.17 \text{ mi}^2
17. 5 \text{ ft}

**Reteach**
1. 3.75
2. 0.8\overline{3}
3. 3.\overline{6}
4. 9, −9
5. 7, −7
6. \( \frac{5}{6}, -\frac{5}{6} \)
7. 3
8. 5
9. 9

**Reading Strategies**
1. Yes, because it can be written as a fraction: \( 0.62 = \frac{62}{100} = \frac{31}{50} \).
2. No, because it cannot be written as a decimal that terminates or repeats.
3. Yes, as long as the decimal is infinite and nonrepeating, such as 0.31311311131111….
4. Yes, for example, \( \frac{2}{3} = 0.\overline{6} \).
5. A decimal that is an irrational number is infinite and nonrepeating, such as the value for \( \pi \).
6. Both are real numbers and both can be written as decimals.

**Success for English Learners**
1. \( \frac{1}{4} = 0.25 \)
2. Possible answer: If you have a square with a side length of 5, then \( 5^2 \) is how you find the area of that square.
3. Because the answer is an approximation.

**LESSON 1-2**

**Practice and Problem Solving: A/B**
1. real, rational
2. real, irrational
3. real, rational, integer
4. real, rational, integer
5. real, rational
6. real, rational, integer, whole
7. false; irrational real numbers include nonterminating decimals
8. true; Integers include all whole numbers and their opposites.
9. rational; all money amounts can be written as fractions
10. real numbers; the temperature can be any number between 0 and 100 degrees Celsius

**Practice and Problem Solving: C**
1. real, rational, integer
2. real, rational, integer
3. real, rational
4. real, rational, integer, whole
5. real, rational
6. real, rational, integer, whole
7. integers; possible points are positive and negative numbers
8. real; elevation can be any number above or below zero
9. no; there are an infinite number of rational numbers between any two integers
10. 10 will be whole numbers (square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100); 90 will be irrational numbers
11. all of the negative numbers that are not fractions or decimals
12. all fractions, irrational numbers, or decimals less than 0

**Practice and Problem Solving: D**
1. real, rational
2. real, irrational
3. real, rational
4. real, rational, integer
5. real, rational
6. real, rational, integer, whole
7. true; all fractions are rational numbers, and all rational numbers are real numbers
8. false; Negative fractions and decimals are not integers.
9. whole numbers; There are 0 or more people in a movie theater.
10. real numbers; The square root of 1, 4, or 9 would be whole numbers, but the square root of 2, 3, 5, 6, 7, 8, 10, 11, or 12 would be irrational numbers. (Each cube can show 1 through 6, so the sum can be 2 through 12. Of these numbers, 4 and 9 are perfect squares, so their square roots are whole numbers. The other possible sums have square roots that are irrational. The set of numbers that contains whole numbers and irrational numbers is the real numbers.)

---

**Real Numbers**

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>10</th>
<th>Irrational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Integers</strong></td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td><strong>Whole Numbers</strong></td>
<td>-99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td></td>
</tr>
</tbody>
</table>

**Reteach**
1. Yes
2. Yes; \( \sqrt{16} = 4 \), which can be written as \( \frac{4}{1} \).
3. Yes
4. Is it a whole number? Yes.
5. real, rational, integer, whole

**Reading Strategies**
1. rational; irrational
2. fraction; decimal
3. terminating; repeating
4. positive; negative
5. counting
6. a. integers
   b. rational numbers
   c. whole numbers
   d. irrational numbers

**Success for English Learners**
1. real
2. not real
3. real, rational
4. real, rational, integer

**LESSON 1-3**

**Practice and Problem Solving: A/B**
1. <
2. <
3. <
4. >
5. >
6. >
7. Great white, White-angled sulphur, Large orange sulphur, Apricot sulphur
8. Between Large orange sulphur and Apricot sulphur
9. $\frac{\sqrt{7}}{2}, 2, \sqrt{8}$
10. $\pi, \sqrt{12}, 3.5$
11. $-20, \sqrt{26}, \sqrt{35}, 13.5$
12. $-5.25, \frac{3}{2}, \sqrt{6}, 5$
13. $1 + \frac{\pi}{2}, \frac{5}{2}, \sqrt{12} - 1, 2.25$

**Practice and Problem Solving: C**

1. a. $2\pi - 1.68; \sqrt{13} + 1; 4.6$
   
   b. Yes, if you used $\pi \approx \frac{22}{7}$, then $2\pi - 1.68 \approx 4.6057142$, which would be greater than $\sqrt{13} + 1 \approx 4.6055512$, so from least to greatest the order would be $\sqrt{13} + 1, 2\pi - 1.68$, and $4.6$.

2. a. $\frac{\sqrt{23}}{2} - 1 \approx 1.40; 1 + \frac{\pi}{9} \approx 1.35$
   
   $\sqrt{12} - 2.2 \approx 1.26; 1.18$

   b. $\sqrt{12} - 2.2, \frac{\sqrt{23}}{2} - 1$

   c. Justin’s shelf is suitable to use in the closet. The other shelves are too long. Justin’s shelf is shorter than the closet width, but it can be held up using the 3.5-centimeter brackets.

**Practice and Problem Solving: D**

1. $<$
2. $>$
3. $<$
4. $>$
5. $>$

**Reteach**

1. $\sqrt{8}, \pi, 4$
2. $5, \pi + 2, \frac{17}{3}$
3. $-2, \sqrt{2}, 1.7$
4. $\frac{3}{2}, \sqrt{5}, 2.5$
5. $\sqrt{13}, 3.7, \pi + 1$
6. $\frac{\sqrt{5}}{2}, \pi - 2, \frac{5}{4}$

**Reading Strategies**

1. The square root of thirteen is less than four.
2. Five-hundred-and-one thousandths is approximately one half.
3. The square root of 25 equals 5.
4. Pi plus one is greater than two thirds. OR The sum of pi and one is greater than two thirds.

5. $\frac{18}{2} = 9$
6. $5.17 > \sqrt{23}$
7. $\frac{2}{3} < \pi$
Success for English Learners
1. <
2. $\pi + 1, \sqrt{19}, 4.4$
3. Answers may vary. Sample answer: when comparing runners’ marathon times

MODULE 1 Challenge
1. No square numbers are prime.
   No $S$ is $P$.
2. Some prime numbers are odd.
   Some $P$ is $O$.
3. All even numbers are integers.
   All $E$ is $I$.
4. $S \cap O$
5. $P \cap S$
6. $I \cap S$
7. $O \cap P$

MODULE 2 Exponents and Scientific Notation
LESSON 2-1
Practice and Problem Solving: A/B
1. 125
2. $\frac{1}{49}$

LESSON 2-1
Practice and Problem Solving: A/B
1. 125
2. $\frac{1}{49}$

Practice and Problem Solving: C
1. 1024
2. $\frac{1}{4}$
3. $\frac{2}{5}$
4. 10,000
5. $\frac{1}{144}$
6. 2048
7. 1; Explanations will vary. Sample answer: If you add all the exponents, you get $n - 21$ and when $n = 3, 7n - 21 = 0; 2^0 = 1$.
8. Predictions will vary.
9.  

<table>
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<th>Number of Folds</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Rectangles</td>
<td>$2^0, 1$</td>
<td>$2^1, 2$</td>
<td>$2^2, 4$</td>
<td>$2^3, 8$</td>
<td>$2^4, 16$</td>
<td>$2^5, 32$</td>
<td>$2^6, 64$</td>
<td>$2^7, 128$</td>
<td>$2^8, 256$</td>
<td>$2^9, 512$</td>
<td>$2^{10}, 1,024$</td>
</tr>
</tbody>
</table>

10. Answers will vary.

11. Predictions will vary.

12. Answers will vary. Sample answer: Raise 2 to the power of the cut to see how many pieces. On the 5th cut, $2^5 = 32$ pieces.

**Practice and Problem Solving: D**

1. \[
\frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}
\]

2. $6 \times 6 = 36$

3. $3 \times 3 \times 3 \times 3 \times 3 = 243$

4. 1

5. \[
\frac{1}{7 \times 7} = \frac{1}{49}
\]

6. $10 \times 10 \times 10 \times 10 \times 10 = 100,000$

7. \[
\frac{(3 \cdot 2)^6}{(7 - 1)^4} = \frac{6^6}{6^4} = 6^{6-4} = 6^2 = 36
\]

8. $(3^2 \cdot 3^1) = 3^{2+1} = 3^3 = 27$

9. $4^2 \cdot 4^3 = 4^{2+3} = 4^5 = 1024$

10. $(4^2)^3 = 4^{2 \cdot 3} = 4^6 = 4096$

11. $(4 - 3)^2 \cdot (5 \cdot 4)^0 = 1^2 \cdot 20^0 = 1 \cdot 1 = 1$

12. $(2 + 3)^5 \div (5^2) = 5^5 \div 5^4 = 5^{5-4} = 5^1 = 5$

13. $(2^3)^2 = 2^6 = 64; (2^3)^2 = 2^6 = 64; $Both are equal. Explanations will vary.

**Reteach**

1. subtract; 27

2. add; \( \frac{1}{8} \)

3. multiply; 729

4. add; 625

5. subtract; \( \frac{1}{16} \)

6. multiply; 1296

**Reading Strategies**

1. The exponent decreases by 1 as you move down the column.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^3 = 125$</td>
<td>$6^3 = 216$</td>
<td>$10^3 = 1000$</td>
</tr>
<tr>
<td>$5^2 = 25$</td>
<td>$6^2 = 36$</td>
<td>$10^2 = 100$</td>
</tr>
<tr>
<td>$5^1 = 5$</td>
<td>$6^1 = 6$</td>
<td>$10^1 = 10$</td>
</tr>
<tr>
<td>$5^0 = 1$</td>
<td>$6^0 = 1$</td>
<td>$10^0 = 1$</td>
</tr>
<tr>
<td>$5^{-1} = \frac{1}{5}$</td>
<td>$6^{-1} = \frac{1}{6}$</td>
<td>$10^{-1} = \frac{1}{10}$</td>
</tr>
<tr>
<td>$5^{-2} = \frac{1}{25}$</td>
<td>$6^{-2} = \frac{1}{36}$</td>
<td>$10^{-2} = \frac{1}{100}$</td>
</tr>
</tbody>
</table>

**Success for English Learners**

1. Answers will vary. Sample answer: A negative exponent tells you how many times you use the base as a divisor.

2. 9

3. 64

4. $x^{20}$

5. 4096

6. 12

7. $z^{12}$

**LESSON 2-2**

**Practice and Problem Solving: A/B**

1. $10^2$

2. $10^4$

3. $10^5$
4. $10^7$
5. $10^6$
6. $10^3$
7. $10^9$
8. $10^0$
9. 1000
10. 100,000
11. 10
12. 1,000,000
13. 100
14. 1
15. 10,000
16. 10,000,000
17. $2.5 \times 10^3$
18. $3 \times 10^2$
19. $4.73 \times 10^4$
20. $2.4 \times 10^1$
21. $1.4565 \times 10^4$
22. $7.001 \times 10^3$
23. $1.905 \times 10^7$
24. $3.3 \times 10^1$
25. 6000
26. 450
27. 70,000,000
28. 10,500
29. 3052
30. 5
31. 98.7
32. 54.3
33. $3.844 \times 10^5$ km
34. 6380 km

**Practice and Problem Solving: C**

1. $2500 < 2,500,000$
2. $5,000,000 > 2,500,000$
3. $3 < 10$
4. $4025 < 10,250$
5. $4.50 \times 10^2; 1.2 \times 10^3$
6. $2.3 \times 10^5; 3.2 \times 10^4$
7. $2.35 \times 10^5; 3.25 \times 10^5; 5.32 \times 10^5; 2.35 \times 10^6; 3.25 \times 10^6; 5.32 \times 10^6$
8. $0 \times 10^0; 1 \times 10^0; 5 \times 10^0; 1 \times 10^1; 5 \times 10^1$
9. False; $1 \times 10^3 \text{ m} = 1 \times 10^3 \times 10^1 \text{ cm} = 1 \times 10^4 \text{ cm}$, which is less than $1 \times 10^6 \text{ cm}$
10. False; $9 \times 10^1 \text{ m} \times 9 \times 10^1 \times 10^3 \text{ mm} = 9 \times 10^4 \text{ mm}$, which is not equal to $9 \times 10^1 \text{ mm}$
11. $15,000 - 9,000 = 6,000$; 6,000 tickets are available
12. $1.5 \times 10^4 = 15,000$, and $15,000 > 2500$, so the athletic team is more popular

**Practice and Problem Solving: D**

1. 100
2. 100,000
3. 10,000
4. 1000
5. $10 \times 10 \times 10 \times 10 \times 10$
6. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
7. $10 \times 10 \times 10 \times 10 \times 10 \times 10$
8. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
9. $10^3$
10. $10^1$
11. $10^5$
12. 10
13. 1000
14. 10,000
15. $1,000,000,000$
16. 100,000
17. 1
18. 3
19. 2
20. 6
21. 0
22. 3560
23. 9000
24. 68,750
25. 4,005,000
26. 1000 ft$^3$

**Reteach**

1. 3.46; 4; $3.46 \times 10^4$
2. 1.0502; 6; $1.0502 \times 10^6$
3. 1057
4. 300,000,000
5. 524,000

**Reading Strategies**
1. 2; 2.95; 2; 2.95 \times 10^2
2. 1; 1.05; 4; 1.05 \times 10^4
3. 4; 4.505; 6; 4.505 \times 10^6
4. between 2 and 9; 4; 25,000
5. after the 7; 5; 700,000
6. between 1 and 2; 3; 1234

**Success for English Learners**
1. 3.28 \times 10^5 > 3.28 \times 10^3 because 
   3.28 \times 10^5 = 328,000 and 
   3.28 \times 10^3 = 3280.
2. A car; 3 \times 10^6 = 3,000,000 g, or 3,000 kg, 
   and a car would weigh many kilograms 
   but a hair would not.
3. 1.86 \times 10^5
4. 4,567.89

**LESSON 2-3**

**Practice and Problem Solving: A/B**
1. \(10^{-2}\)
2. \(10^{-4}\)
3. \(10^{-5}\)
4. \(10^{-7}\)
5. \(10^{-6}\)
6. \(10^{-3}\)
7. \(10^{-9}\)
8. \(10^{-1}\)
9. 0.001
10. 0.00001
11. 0.1
12. 0.000001
13. 0.01
14. 0.00000001
15. 0.0001
16. 0.000001
17. \(2.5 \times 10^{-2}\)
18. \(3 \times 10^{-1}\)
19. \(4.73 \times 10^{-4}\)

**Practice and Problem Solving: C**
1. 0.0052; 0.0000052; >
2. 0.000005; 0.000205; <
3. 3; 0.1; >
4. 0.00502; 0.000205; >
5. \(1.2 \times 10^{-3}; 4.5 \times 10^{-1}\)
6. \(3.2 \times 10^{-4}; 2.3 \times 10^{-6}\)
7. \(2.35 \times 10^{-6}, 3.25 \times 10^{-6}, 5.32 \times 10^{-6}, 2.35 \times 10^{-5}, 3.25 \times 10^{-5}, 5.32 \times 10^{-5}\)
8. \(0 \times 10^0, 1 \times 10^{-1}, 5 \times 10^{-1}, 1 \times 10^0, 5 \times 10^0\)
9. 1 \times 10^{-3} m = 1 mm; 1 \times 10^{-1} cm = 1 mm; 
   equal, so false
10. 7 \times 10^{-1} cm = 7 mm; 7 \times 10^{-3} m = 7 mm; 
    equal, so false
11. 3.5 \times 10^{-1} cm = 3.5 mm; 3.5 \times 10^{-3} m = 3.5 mm; 
    equal, so true
12. 9 \times 10^{-1} mm; 9 \times 10^{-4} m = 9 \times 10^{-1} mm; 
    equal, so true
13. 9 \times 10^{-3} liters = 0.009 liters; 3 \times 10^{-6} liters 
    = 0.000003; 0.009 \div 0.00003 = 300 drops
14. 0.00025 meters = 2.5 \times 10^{-4}; 0.000125 meters = 1.25 \times 10^{-4}; area = length \times width = (2.5 \times 10^{-4}) \times (1.25 \times 10^{-4}) = 3.125 \times 10^{-8} square meters
Practice and Problem Solving: D

1. \( \frac{1}{100} \)
2. \( \frac{1}{100,000} \)
3. \( \frac{1}{10,000} \)
4. \( \frac{1}{1000} \)
5. \( \frac{1}{10 \times 10 \times 10 \times 10 \times 10} \)
6. \( \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10} \)
7. \( \frac{1}{10 \times 10 \times 10 \times 10} \)
8. \( \frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10} \)
9. \( \frac{1}{10^3} = 10^{-3} \)
10. \( \frac{1}{10} = 10^{-1} \)
11. \( \frac{1}{10^2} = 10^{-2} \)
12. \( \frac{1}{10^4} = 10^{-4} \)
13. \( \frac{1}{10} \)
14. \( \frac{1}{1000} \)
15. \( \frac{1}{10,000} \)
16. \( \frac{1}{1,000,000,000} \)
17. \( \frac{1}{100,000} \)
18. \( \frac{1}{1,000,000,000,000} \)
19. −4
20. −1
21. −8
22. −3
23. 0.00356

Reteach
1. \( 2.79 \times 10^{-2} \)
2. \( 7.1 \times 10^{-5} \)
3. \( 5.06 \times 10^{-7} \)
4. 0.000235
5. 0.0065
6. 0.0000707

Reading Strategies
1. 1; 3; right; −3; \( 1.23 \times 10^{-3} \)
2. 5; 6; right; −6; \( 5.67 \times 10^{-6} \)
3. −8; left; 8; 0.000000067
4. −4; left; 4; 0.000321

Success for English Learners
1. \( 5.75 \times 10^{-3} > 5.75 \times 10^{-4} \) because \( 5.75 \times 10^{-3} = 0.00575 \) and \( 5.75 \times 10^{-4} = 0.000575 \).
2. A hair; \( 3 \times 10^{-7} = 0.0000003 \text{ g} \), and a hair would have a mass that is less than a gram, but a bicycle would not.
3. \( 4.93 \times 10^{-4} \)
4. 0.0000321

LESSON 2-4

Practice and Problem Solving: A/B
1. \( 2.79 \times 10^4 \)
2. \( 3.83 \times 10^6 \)
3. \( 6.67 \times 10^9 \)
4. \( 4.48 \times 10^4 \)
5. \( 4.16 \times 10^{17} \)
6. \( 2.0 \times 10^3 \)
7. \( 8.85 \times 10^{10} \)
8. \( 6.0 \times 10^7 \)
9. \( 4.1E + 4 \)
10. \( 9.4E - 6 \)
11. \( 5.2 \times 10^{-6} \)
12. \( 8.3 \times 10^2 \)
24. 0.00009
25. 0.0006875
26. 0.000004005
27. 0.15 ft²; \( 1.5 \times 10^{-1} \)
13. $7.0 \times 10^4$
14. $1.4 \times 10^4$
15. $1.6 \times 10^4$
16. about $3.0 \times 10^4$, or about 30,000 strides

**Practice and Problem Solving: C**
1. $3.574 \times 10^4$
2. $5.416 \times 10^4$
3. $1.6897 \times 10^7$
4. $7.08 \times 10^3$
5. $2.496 \times 10^{13}$
6. $7.0 \times 10^{12}$
7. $2.89 \times 10^{-2}$
8. $2.3 \times 10^{-4}$
9. $6.0 \times 10^{-2}$
10. $3.65 \times 10^2$
11. $3.15 \times 10^8$
12. $6.8985 \times 10^8$ kilowatt hours; 6,898,500,000 kilowatt hours
13. 459,900 households

**Practice and Problem Solving: D**
1. $4.044 \times 10^4$
2. $1.028 \times 10^4$
3. $2.8 \times 10^6$
4. $5.65 \times 10^4$
5. $2.048 \times 10^{13}$
6. $1.92 \times 10^1$
7. $1.025 \times 10^9$
8. $2.0 \times 10^8$
9. $3.3 \times 10^{-3}$, $6.9 \times 10^5$
10. $7.1E + 5$, $4.4E - 3$
11. $1.0 \times 10^6$
12. $\frac{1.0 \times 10^6}{8.64 \times 10^4} \approx 0.116 \times 10^2$ or 1.16
    $\times 10^1 \approx 11.6$ days

**Reading Strategies**
1. $2.82 \times 10^4$
2. $1.92 \times 10^{10}$
3. $2.0 \times 10^3$
4. $2.0 \times 10^{-1}$
5. $3.07 \times 10^8$
6. $3.29 \times 10^{10}$

**Success for English Learners**
1. Sample answer: When dealing with very large or very small numbers
2. $5.5 \times 10^2$
3. $1.5 \times 10^6$
4. $1.45 \times 10^{10}$
5. $3.74 \times 10^5$
6. $6.12 \times 10^{11}$
7. $5.6 \times 10^4$

**MODULE 2 Challenge**
1. $9,460,528,400,000,000$ m; rounded, this is 9 trillion km
2. $9,460,528,400,000$ km; $9.4605284 \times 10^{12}$ km; almost 10 trillion
3. $(9.4605284 \times 10^{12}$ km)(3.26); $3.08 \times 10^{13}$ km
4. $0.000000001$ m; $1 \times 10^{-9}$ m
5. $(1 \times 10^{-9}$ m)(0.1) = $1 \times 10^{-10}$ m
6. $(1 \times 10^{-9}$ m)(1,000) = $1 \times 10^{-6}$ m
7. $\frac{3.08 \times 10^{16}}{1.0 \times 10^{-9}} = 3.08 \times 10^{25}$; 1 parsec is about $3 \times 10^{25}$ nanometers